REWIRING YOUR MATH KNOWLEDGE

Real World Examples for Rational Numbers

Rational numbers represent just a different quantity, different than integers. **A rational number is a ratio of two integers**. It is very useful to point that very early, that a rational number, or a rational quantity, is just a quotient **of two integers** (where divisor is different than 0), or two whole numbers, because kids are already familiar with integers. Rational numbers are quantities that sit between integers, although, strictly speaking integers are a subset of rational numbers, because every integer divided by 1 is by definition a rational number as well. Set of integers is a subset of rational numbers. But, for the introduction it should be acceptable to show that rational numbers are quantities that are between integers. For instance 7/5, 3/8, 17/19, 123/127, are all rational quantities. Note, however, that they are just constructed as division of two integers, 7, 5, 3, 8, 17, 19, 123, 127. Hence any **quotient**, any **ratio** of m/n (where m, n belong to the set of whole numbers or integers, n is different than 0) is a rational number.

There is another way to nicely explain introduction of rational numbers. **You really only have to know integers to construct all rational numbers**. Let's say number 37/258. How you get this number using only integers? Let's use a length unit of measure, say, 1 meter. Take one meter and divide it carefully to 258 segments (obviously you can divide it to any number of segments you wish). Take that one, small segment which is one 258th part of meter and multiply it, or, put one after another, 37 times. The length you get by adding these small segments is exactly 37/258 meters! Note that we used **only integers** to obtain this new number, new length. You can call these kinds of lengths as you wish, but someone agreed to use word "rational" because it is written as a **ratio** of two integers. **But, what this "ratio" really tells you is that you use known numbers, integers, and do operations, first division and then multiplication to obtain the new length.**

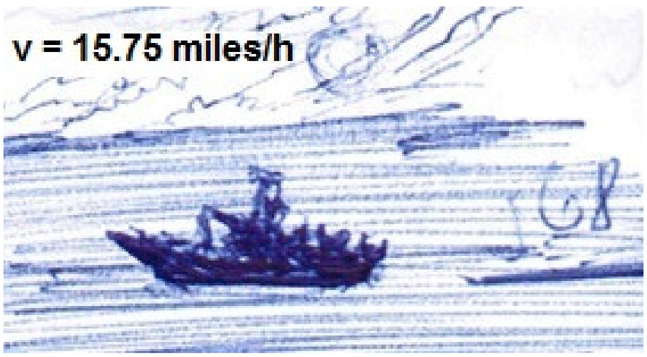
Motivation for mathematics, increased focus of a student, and plenty of examples for whole numbers, rational and real numbers can be made by using:

- Pirates, pirate's ships. Invent a story of a pirate's ship coming to an island. Wind speed is **10.754** m/s, rum bottle is **3/4** full, surface area of sail is **25.78** square meters.

- Scuba diving. Various divers are at various depths, **7.89** meters, **12.75** m, and **21.997** m. Oxygen can last **1.23** hrs for each diver, etc. Temperature of the water is **12.79** degrees Celsius.

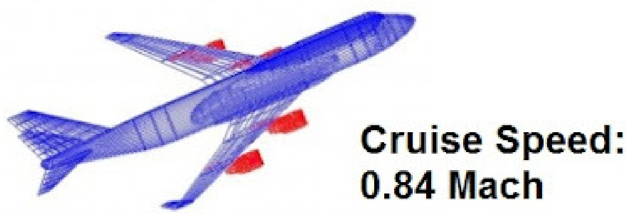
- Text messaging. Each text message costs **$0.87**. How much **127** messages cost?
- Car racing. Speed of Formula 1 cars (**256.78 km/hr**), time of arrival, fuel consumption (**72.59 L/km**), engine temperature (**985.23 C**), laps counts **(2.5)**, tire rubber temperature, pit time(**58.5** sec), randomness of pit times (**probability distribution, average, expectation**), track length (**10.25**km), compare tire diameter, volume with the length of the track.

- Scale models airplanes. If the scale is 1:80, what is the wing span on the model, if the real wing span is 57.52m?
- Duration of phone calls (**3.25** min).
- Pets and other animals. How much food they drink and eat (**150.75 g of raw meat**).
- Volume of ice cream, ice cream cones (**1/3 of a cone**).
- Space travel (**duration, distance, speed**).
- Food distribution (airplanes, countries, how many people, food ratios, nutrition value). Kids and everyone like to help. Good motivation to use mathematics.
- Astronomy
- Speed in general, a numerical value, obtained by measuring distance and time.



- Weather, air temperature, wind speed and direction, nature.

- Favorite student's rock star contact lenses dimensions
- Airplane flight deck instruments.
- Car driving instruments, dashboard.
- Show the graph of stock prices on NYMEX web page
- Airplanes, trains, ships, transport.



Music, musical instruments, tones, songs

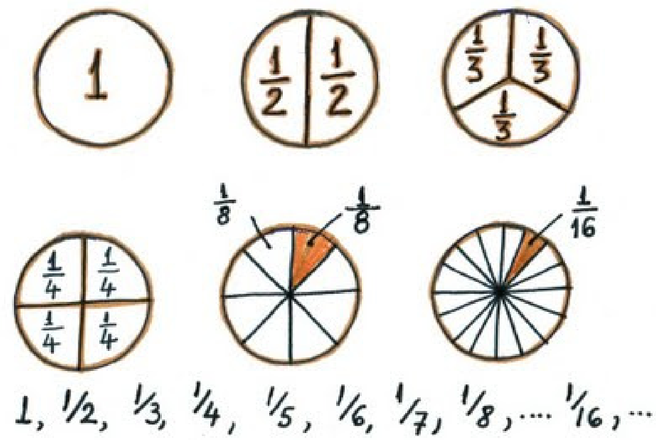
- Surface areas of squares and rectangles
- DVD, CD surface areas
- TV screen surface areas calculations
- Liquid level in the bottle, or any container
- Time measurements by watches.

The fact is, mathematics doesn't need real life examples to illustrate its concepts. Take, for example, vampires! Twilight Saga is very popular now, so why don't use vampires to illustrate rational numbers. If a vampire sucks 278.52 mL of blood per 1 minute, how much blood a vampire can suck out after 7 minutes? You notice, real life examples are not necessary to illustrate rational numbers, and for that matter, many of other concepts in mathematics. We can count real world objects as well as completely fictional ones. Math doesn't care where the numbers are coming from. Math can't tell real world from fictional one! Another example. If Harry Potter flies 10 m/s how many meters he will advance after flying 5 seconds? Harry Potter and his flying device are fictional, yet mathematics is very real.

During explanation make every attempt to clearly **separate non mathematical reasoning from actual mathematical calculations** in each of these examples. Clearly show that real world situations can be a motivation to count and measure certain objects, and hence to obtain certain numbers (rational, integers), but emphasize that those numbers and operations on them can exist by themselves too (pure math). Also, show that you can go from math to real world. You can say that you want to fill 1/2 of a bottle with water. Note that, here, you first came up with a number 1/2. You don't know yet what this quantity will represent! It is only a number 1/2. Only in the next, separate step, you decide it will be related to a bottle of water. Bottle of water! Not 1/2 of a watermelon, or 1/2 of a cake, or 1/2 of an apple, but a 1/2 of bottle of water!

One of the best examples to illustrate rational numbers may be to start with pizza.

Here is one whole pizza. That's number one. But, look. You can have half a pizza, right? How many halves will make one whole pizza? 2! How about one quarter of a pizza? How many quarters will make one whole pizza? 4! So, the **main question** we are asking here is what is that **quantity** then when **multiply** by 4 will give the whole **one**. That's 1/4! What is the quantity that when multiply by 2 will give one whole pizza? It's 1/2!



Let's look at that pizza again. How many friends can divide the pizza so each one can have an equal slice? 2? 3? More? Sure, say 10 friends. Even more? Of course, there can be 120 people wanting to get a slice of ONE pizza. Each one of them will get exactly 1/120th part of pizza. You can see that we can divide pizza to as many slices as we want. What would be other numbers to divide pizza in equal parts? We surely can give examples of 1/9, 1/12, 1/125, 1/1279, 1/100242, ...Any integer can be a divisor!

Now, let's say you want to divide 2 pizzas to, say, 15 friends. Each one will get, in that case, 2/15 part of pizza. Maybe you are having a big party and you have 5 pizzas and 17 friends. Each one will get 5/17 of a pizzas. This **quantity**, 5/17, is the real quantity, it exists as a real slice (how you are going to cut pizza equally is a technical issue :-). So, we can see now, that dividend, i.e. how many pizzas you can have can be any integer. You can have 1, 2, 5, 15, ...20, 100, pizzas. And you can divide them between 2, 3, 120, ...150, friends. This tells us that rational number can be represented by two integers, one is dividend, another is divisor. Like this a/b = c, where "a" can be any integer, "b" can be any integer. Number "c" is their quotient.

Another interesting example to illustrate rational numbers is when you want to divide juices from 5 one liters juice boxes or more to 17 friends, but each one is getting a (unusual size) glass of 107 ml. You will start filling up glasses from the first box, and you can show your student how sometimes when a box is empty, and the glass partially filled then the glass must be filled up from another juice box or bottle. That's the nice point to introduce rational numbers too.

Let me demonstrate you, in one example, what is the difference between **pure** and **applied math**. It will help to understand where the numbers come from and why they can be used in real life situations, and how **mathematics can be independent discipline** no matter how many real life examples are there. Moreover, we should not move from integers to rational numbers without having another, perhaps deeper look, what numbers are, and their relation to objects counted. Once this is clear, then, it will be way more clear how we define ANY kind of a number. To give you an advanced start, all numbers are constructed from integers, either as ratios of two integers, or, sums of ratios of two integers, i.e. sums of fractions, finite or infinite (with repeating or non repeating decimals).

When you see a person writing down 2 + 3 =... you don't know what objects she might be adding! But you know the result will be 5. That's, actually, pure math! **You see, a person is writing down 2 + 3 =… Now, let's say it again, you don't know what that person has in mind, which objects she was counting. But, you know that the result will be 5! That's, so called, pure math. You have just abstracted pure numbers from any objects whose counts these numbers can represent!** While objects counted (whatever they may be) can have color, weight, temperature, texture, taste (if you count apples), the numbers you have just dealt with have their own properties, which are magnitudes and their relations with other numbers, i.e. great than, less than, equal, divisible by, etc. But these are numbers' properties and not the properties of objects we have counted. Investigating these numbers' properties is subject of pure math.

One more nice point about pure math. You don't need to see 5 objects to come up with the number 5! You can start with pure number 1 and add four times number 1 and you got 5! You see how even you can abstract operation of addition from any real world object that may be counted. You can deal with number 1 only and by adding (or subtracting number 1) you can define all integers, without even counting any real world object.

In the same way you can construct rational numbers, by dividing integers you just obtained! The examples in real world comes at the point when you associate to a number a thing or object you have counted! So, it is true that you can obtain a number, count, in two ways, by counting real world objects or just by declaring the number you are interested in, because you just constructed them in the field of pure math!

Once you are aware that you can work ONLY with numbers, or counts, you are in a position to investigate properties of these counts, without considering at all where they come from. Note, though, when you investigate properties of numbers, no real world objects, or examples, enter discussion. You may now want to continue to develop math as a separate discipline! Make more examples, think of more numbers, operations on them! 5 + 3 = 8. 3 x 4 = 12. 8 - 5 = 3. You can say now, that if you have 7 and you add 8 you will get 15, just by dealing with numbers, no matter what objects and rules about them were involved. You realize that you can work with numbers only! And with operations you have at your disposal, addition, subtraction, division, multiplication. Now, be sure, there are no other operations in math. These fours are sufficient to define all "kinds" of numbers and all mathematics. Other, so called, more complex operations (although they are not so in many cases) are just different sequences of these basic four, +, -, x, /, and mathematicians give them exotic names, that's all.

This is probably the most important step you have made in learning math so far.

Rational numbers can be quickly introduced to kids. The issue may be that kids are asked for too long to deal only with integers, that they may think there are no other numbers, or, that the other numbers must be difficult to understand since their introduction is so much postponed.

The major concept to explain to kids is a QUANTITY. Quantity is what we deal with in mathematics. Make no mistake, as we have shown, mathematics can exist without referencing any real world examples. It is because math deals ONLY WITH NUMBERS, COUNTS. However, math can be used in real life once you start **keeping track** what is counted and why.

The confusing thing for kids is that they continuously try to link the ways HOW the counts are obtained and why, with mathematics, thinking that the description HOW and WHY counts are obtained is a part of mathematics. But, it is not so! Mathematics knows only about numbers, it does not care where they come from. Think about it. It is you who will keep track how, what, when, and why you have counted certain objects.

You can invent the game for two kids. One kid will define what to count and when, while the other kid will just write down the numbers, do the required operations on them and tell the result back to the first kid. The second kid is pure mathematician. The first kid is applied mathematician. This is how the thinking about math and applied math should go. Second kid, the "calculator" kid, will realize that the same operations and results can be applied and reused for many different requests and objects defined by the first kid.

Students should be shown that all the other numbers, rational, real, imaginary, transcendent, irrational, are CONSTRUCTED from integers. Students should not be under impression that somehow rational or irrational numbers are complicated or exotic. It should be shown that rational numbers are fractions, and fractions are made of TWO INTEGERS. That's it! It's the operation on integers, in this case their division, that led to new so called type of numbers. But, these are not new "type" of numbers, these are just new, different quantities DEFINED IN A NEW WAY. That should be clear. It is operations (division) that has been applied to two integers to obtain a new quantity, somewhere in between two integers. Later, you can show that irrational numbers, the numbers that can not be represented by ratios of two integers, actually CAN be represented by a sum (though infinite) of smaller and smaller fractions! You see, using rational numbers, fractions, and adding them, you can get rational numbers again, but, also you can get irrational numbers, like square root of 2, which is an infinite sum of smaller and smaller fractions. Note about infinite sums here. You may think at first that when you infinitely add quantities, the sum must be infinitely big. But it is not so if the quantities you add up are smaller and smaller fractions! Just try to add more and more elements 1/(2*(2^n)), like 1/2 + 1/4 + 1/8 + 1/16 + 1/32 +... Even if you use a calculator you will see that the sum is closer and closer to ONE, 1, as you add more and more smaller fractions! That's the essence of a irrational numbers! As you can see, knowing integers can greatly help understanding whole number system! Using integers you constructed rational numbers, as fractions of two integers, and then, on one elegant idea, you obtained irrational numbers by adding up many smaller and smaller rational numbers. Knowing this you are absolutely ready to procede further in mathematics very fast.

As I mentioned, an example of irrational number was a square root of 2. Here is another fanous example of an irrational number, Pi:

3.14159265358979323846264338327950288419

7...

Note that rational number can be represented as a ratio of two integers and as a number with finite decimal places or with infinite but REPEATING decimals, like 0.33333333.... which is 1/3. Irrational number can not be represented as a ratio of two integers. Irrational number can be represented as a number with infinite but NON-REPEATING decimals. Examples are square root of 2 and Pi.

At this point I would like to show you how mathematics can be an independent discipline and also can be, and is, used in many applications and disciplines in real life. I will make a comparison between lines, some shapes and numbers. When you look at the building, and you want to draw it, you will draw some lines, squares, rectangles, trying to mimic the shape of the building as truthfully as possible. Look at the lines you draw. You draw vertical, horizontal, and lines at any arbitrary angle. It appears that which line and where you will draw it will depend on the shape of the building. And that's true. Now, look away from the building and get a new, blank sheet of paper. Draw a line on it. Draw horizontal lines, vertical, and at arbitrary angles. You are not required to draw a building. Just draw lines. You see how you abstracted lines, and, moreover you can play with them without taking care whether they represent anything in real world. Moreover, you can draw completely new building with your lines and call a company to actually build a new object from your drawings!

Similar things is with numbers. You can count real objects and get their numbers and deal with them. You can add, subtract, multiply numbers following the real world examples, like, "how many litres of gas I will use if I travel 125 km with the car that consumes 8.9L/km..." etc. When you calculate this or any other example, you deal only with numbers. You are the one who will keep track of units, what you have counted. But, then, you can notice that 7 x 5 = 35 regardless what is counted! Similar thing happened with lines! You could draw lines without worrying if they represent any object in real world. Now, you can play with numbers, any number, and use ANY operations on them without worrying do they currently represent anything in real world. That's the essence of math. When you do applied math you still do pure calculations while keeping aside the units, what you have counted. But, in pure math you start with some numbers, it's up to you with which ones, without providing reasons why they are there, and do calculations on them. Try both scenarios and you will see the point!

Asking someone to find a real life example of a number (integer, rational, etc) is the same as asking to find a real world example for a straight line.

In the same way a straight line is abstracted from all real world straight distances, a number is abstracted from all the objects that have the same count.

Looking at the line only one can not say what it represents but it he can say what line may represent. The same thing is a number. Looking at the number, one can not say what does it represent, but only what it may represent.